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Difficulties of concept of function: The case of general secondary school students of Ethiopia

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ABSTRACT - This study presents investigation of difficulties of grade nine students have on the concept of function using a descriptive survey method. Both quantitative and qualitative research approaches were used to explore the students' challenges in this study. Data was collected using test, interview and teachers' questionnaire. The four commonly used ways to represent function is then used to analyse students' interpretation and manipulation in different contexts. The finding of the study reviled that students have difficulties in basic definition of function, to identify the difference between function and relation, difficulty to identify an equation with two variables x and y as a function or not. The students have also difficulty in verbal and graphical representation of function. Identified students misunderstanding about the function concept are: any functions should contain both x and y variables, any continuous graph are a function and a discontinuous graph is not a function. This could be because of students' attention to the lesson, their background weakness and students limited English language skills. Based on the findings of the study different recommendations are suggested in order to solve the problems.

Keywords - Concept, difficulties, function, General secondary school, Mathematics.

1 INTRODUCTION

IN daily life of human activities the knowledge of mathematics is important. In line to this Igbokwe [1] underlines as without mathematics there will be no science and without science there will be no technology, and without technology there will be no modern society. The function concept is one of the central concepts underlying mathematics [2]. It provides a basis for high school mathematics courses as well as college courses.

In order to improve students' knowledge of functions, the National Council of Teachers of Mathematics (NCTM) emphasized that student as early as grades 3-5 need to begin looking at representing and analyzing functions using words, tables, and graphs and teachers should also emphasize the importance of using and interpreting several representations, while working with functions, throughout the student's mathematical education [3].

In line with this the objectives of second cycle primary education (Grades 5 - 8) of Ethiopian mathematics curriculum stresses as students should be able to: - understand the notion "function" [4].

This emphasis should help students to develop a list of many different types of functions and their respective representations as they progress through their middle school and high school mathematics courses. They should be also able to manipulate and interpret a variety of functions using several different representations, graphical, tabular, verbal, and symbolic [3], [5].

Even though it is fundamental to mathematics and emphasis is given in all level, many students hold primitive understandings and firmly rooted misconceptions [6], [7]. Tall [8] indicated that the concept of function is a central one in mathematics and high school and college students have shown difficulty in understanding the concept of function. Similarly, Breidenbach et al. [9], indicated that college and high school students do not have a well developed understanding of the concept of function. Students also possess a variety of misconceptions and beliefs that range from continuity issues to conflicts stemming from the representations of functions [10], [11].

Vinner [12] used the constructs of concept images and concept definitions to analyze students' understandings and misconceptions of the function concept. According to him a concept definition is a verbal description of a mathematical concept that accurately describes the concept in a non-circular way; whereas, a concept image is the mental picture that is associated with the concept name in a student's mind. In line with this, Vinner and Dreyfus [13] found that students compartmentalize their concept images and a concept definition, that is; students do not always connect a formal definition to their mental images and this can lead them to misconceptions between the concept definition and one's concept image. Due to this students experience a difficulties and misunderstanding on the conception of function.

Students' think of concept images often focus on a single image or piece of information about the function concept that allows the student to answer a particular mathematics question without consulting the concept definition. Consequently, this partial use of the concept image prevents the development of conceptual understanding [13]. These misconceptions about the function concept may occur for several reasons. For example, according to Vinner & Dreyfus [13], students may not fully understand the formal definition of function.

The concept of function is very complex for several reasons [14]. Firstly, there are many common ways to represent functions, including graphs, formulas, tables, mappings, and descriptions. Secondly, the notion of function involves many other concepts. A few of the sub-concepts associated with it are domain, range, inverse, and composition. Thirdly, there are several accepted definitions

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for function (e.g., dependence relation, rule, mapping, and set of ordered-pairs). Meaningful understanding requires individuals to construct multiple representations as well as operations for transforming from one representation to another.

In addition to this, from researcher's teaching experience students' difficulties in understanding the concept of function were observed. For instance, students had difficulty to accept zero polynomial function (i.e. f(x) = 0) by the association zero = nothing. Similarly, most of high school mathematics teachers complain about the devotion of students in the subject which benchmarks the problem of higher institution student's performance in the science areas. Thus, the purpose of this study is to examine the kinds of difficulties and misunderstanding general secondary school (GSS) students in Ethiopia have with the concept of functions.

2 REVIEW OF THE RELATED LITERATURE

Function is one of the key concepts of mathematics, which can easily be applied to real life situations [11]. Based on study results of [15], [16], function is one of the most important topics in mathematics and it affects the whole mathematics curriculum. The concept of function is central to students' ability to describe relationships of change between variables, explain parameter changes, and interpret and analyze graphs. In line with this, Dreyfus [17] reveled that "it is literally impossible for students to acquire higher level mathematics without understanding the concept of functions'.

2.1 Understanding of the concept of function

The word "understanding" is widely used in discussion about learning and doing mathematics. Understanding in mathematics implies the connections between ideas, facts or procedures [18]. Similarly, as of Clement [19] understanding in mathematics implies an ability to recognize and make use of a mathematical concept in a variety of settings, including some which are not immediately familiar. In making connections, one not only links new mathematical knowledge to prior knowledge but also creates and integrates knowledge structures [20].

According to Tall & Vinner [7]), the concept definition is being the formal mathematical definition, while the concept image is a much wider concept, representing "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes". The independent formation of a concept image and concept definition may be a result of students memorizing a formal definition without connecting meaning to it [12], [21].

Based on study results regarding students' difficulties in understanding the concept of function or misunderstanding occur for several reasons. Lack of understanding a definition may lead to conflicts between students' images and their concept definition [12], [13]. Some students' difficulties in the construction of concepts of function are linked to the restriction of representations when teaching. Researchers pointed out that many students and some prospective teachers do not hold a modern conception of functions [11]. In this regard, Eisenberg and Dreyfus [22] found that mathematics instructors, at the secondary level, traditionally have focused their instruction on the use of algebraic representations of functions rather than the approach of them from the graphical point of view. This is also limit students concept development.

For instance some of students' misconceptions of the concepts of functions reported by [23] are: function is an algebraic term/a formula/an equation; functions should be given by one rule; graphs of functions should be regular and systemic, the constant algebraic form, y = c, c is a constant, is not considered as the representation of a function.

In line with this, Vinner [24] also identifies students believe that a function should be systematic; an arbitrary correspondence is not considered as a function. According to Markovits, Eylon, and Bruckheimer [25] study results, students considered function should be one-to-one and so for example, since $f(x)=12$ is not one-to-one they could not consider it as a function. According to him these students' expectations of functions that may not logically relate to the definition are reasons in their misconceptions of functions.

2.2 Representations of the concept of function

Davis [6] defines representations as "Any mathematical concept, or technique, or strategy – or anything else mathematical that involves either information or some means of processing information – if it is to be present in the mind at all, must be represented in some way." Mathematical concept representation might involve facts about that concept, pictures or procedures we might draw on in order to explore the concept, and how we have felt in the past working with that concept. To understand the concept of function it is important to link all representations of function with its definition, which influence their understanding.

Thus, to be able to link the different representations of function is probably the most important node in the network of students' understanding of the concept of function. Regarding to this scholars Schwarz & Dreyfus, 1995; Sierpinska, 1992 as cited in [26] point out different representations of functions such as; graphs, equations and tables should not be understood as separate entities but as one entity representing a single object - the function.

In his study Thompson [27] suggested to develop a rich concept image, which seems important to develop a rich meaning for and use of the concept, students ought to encounter functions in different representations and make connections between these. In light of this points Hiebert and Carpenter [28] suggested that, to develop students understanding of concepts of function we need to try and access the different connections that a student has through assessment by considering: students' errors, connections

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made between symbols and symbolic procedures and corresponding referents, connections between symbolic procedures and informal problem solving situations and connections made between different symbol systems.

One common misconception in the verbal representations identified by [29] is that: functions exist only if mathematicians give names to different functions (e.g. circles, quadratics, polynomials, and vertical lines) because they have names and equations that can be easily written and identified by the students. Similarly, Sfard [29] found that students believed that all functions can be expressed in a regular manner relating x and y, and that all functions can be expressed by computational formulas. For example, Tall & Bakar [23] found that a majority of secondary school and university students did not regard $y = 4$ as a function because it does not depend on the value of x, but that $x^2 + y^2 = 1$ is a function because it is a familiar.

From studies results of [11], [13], [23] students' algebraic misconceptions come from their perception about function. According to them students consider a function should be given by a rule and equation missing variable is not a function.

Graphical representation is another misconception that students faced in understanding the concept of functions. Jones (2006) as cited in [29] indicated that students often believe that functions are continuous.

Researchers indicated that because of the difficulty of the concept of function students have difficulties in making connections between different representations of the notion (formulas, graphs, diagrams, and word descriptions), in interpreting graphs and manipulating symbols related to functions [31], [32]. Hence as Kabael [33], it is important to use multiple representations (table, graph, formula, procedure, verbal formulation, etc) in the teaching process of the concept of function.

The teacher's role in promoting the students' mathematical activity is crucial. The students' interest will be stimulated by the mathematical tasks selected by the teacher, and by the situations and contexts that the teacher promotes in the class, as well as by their capacity to develop and to lead the students' activity with success. It will be the mathematical tasks and situations that give the opportunity to the students to develop their own algebraic thinking.

3 Research Design

In this study a case-studies method that was descriptive in nature was employed with mixed research approach. A random sampling method was employed to select four GSS of West Showa zone (Ejere preparatory and GSS, Dandi preparatory and GSS, Ambo GSS, and Guder preparatory and GSS). So, the population of this study included grade nine mathematics teachers and grade nine students in these schools.

3.1 Participants

The participants of this study were selected from the sampled schools grade nine students and their teachers. Of 6,370 Grade nine students sample of size 352 were randomly selected using determination of sample size formula

$$n = \frac{Nz^2 p(1-p)}{d^2(N-1) + z^2 p(1-p)}$$

Were; n = required sample size

N = population size

Z = Z statistic for a level of confidence, for the level of confidence of 95%, which is conventional, Z value is 1.96.

P = the proportion (assumed to be .50 since this would provide the maximum sample size).

d = the degree of accuracy expressed as a proportion (.05).

Eleven of twenty six grade nine mathematics teachers were also stratified based on their shift and morning shift teachers were selected. The random number table was used to select sample units from the population. The students were proportionally selected from each four sampled schools randomly. Accordingly, 46 students from Ejere preparatory and GSS, 98 students from Dandi preparatory school, 150 students from Ambo General secondary school, and 58 students from Guder preparatory school were included in the sample who sat for the test on the functions. Of these students who sat for exam, six students were took part in the oral questions interview. The main criterion for selection of the students for the oral questions interview was their willingness to contribute and their achievement in the test questionnaire was also considered. Low achievers (2 students), middle achievers (2 students), and high achievers (2 students) were selected.

3.2 Instrument

In order to get adequate information from the respondents test, interviews, questionnaire, and classroom observation were employed. Test and interview were administered to randomly selected grade 9 students, while the questionnaire administered to randomly selected grade 9 mathematics teachers.

TEST

To examine students' areas of difficulties, paper test was given to them. To determine the students' conceptions of concepts of function and examine their areas of difficulties both closed and open ended paper test was given to them. After the items were selected, prepared and modified evaluation was held by different bodies to check its validity of the test. Moreover, a pilot test were first administered to 30 students in toke GSS in order to determine the difficulty level, discrimination power and reliability of the test items. The reliability of the test was also checked using Kuderrichardson formula 20(KR-20). As a result, the reliability coefficient of the test was found to be 0.79 which is an acceptable level. After making the necessary

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modifications based on the comments of experts and test with very difficult and low discrimination power were revised it was administered to the actual sources of information with the help of assistant invigilators.

ORAL QUESTION

One week after the completion of the paper-pencil test, the oral questions interviews were employed to elicit student understanding of the concept of function. Each interview was tape recorded and lasted about 20 - 30 minutes. The numbers one to six was allocated to the students according to the sequence in which they were interviewed (i.e. S1 and S2 for high achievers, S3 and S4 for middle achievers, and S5 and S6 for low achievers). The interviews were focused on gaining further insight into students' thinking that might help the researcher make sense of their solutions and explanation in the written exam.

QUESTIONNAIRE

The questionnaires for teachers serve to gather information from the teachers regarding to their classroom practices as well as their orientation about students, which helps the researcher to get further information about student difficulties and misunderstanding on functions. In addition it served to triangulate information obtained through test and interview.

3.3 Method of Data Analysis

The raw data collected using test, questionnaire and oral questions were narrated by using mixed research method. The data collected through students' paper-test questionnaire were entered in to the Statistical Package for Social Sciences (SPSS) computer Program presented in tables and quantitatively analyzed, interpreted and reported using frequency and percentages. The data collected through students' oral questions, classroom observations, and open ended students test and teachers'questionnaire were summarized and qualitatively described. Finally, major findings and conclusions were made. Based on the findings, some recommendations have been given.

4 Results and Discussion

4.1 Students' conception of the definition of function

To explore students' conceptions, difficulties and areas of misunderstanding of the concept of function the researcher were asked the students to define a function in sentence and provide examples which represent function and non-function, identify different representation of function and non function with justification on paper test.

Table 1: Summary of Students' response to item 1 (see appendix A)

Responds	Define function	Give correct examples of function	Give wrong examples of function	Don't give example of function	Give correct examples of non function	Give wrong examples of non function	Don't give example of non function
Correctly	58(16.5%)	58(16.5%)	_	_	58(16.5%)	_	_
Wrongly	236(67%)	104(29.6%)	100(28.4%)	32(9.1%)	68(19.3%)	122(34.7%)	46(13.1%)
Missing	58(16.5%)	8(1.7%)	24(6.8%)	26(7.4%)	10(2.8%)	32(9.1%)	16(4.6%)
Total	352(100%)	170(48.3%)	124(35.2%)	58(16.5%)	136(38.6%)	154(43.8%)	62(17.6%)

As presented in table 1, only 58 (16.5%) students defined a function correctly and provided correct examples of function and non-function. On the other hand, among those students who defined wrongly, 236 (67%), about 104(29.6%) of students gave correct examples of function and 68 (19.3%) students constructed correct examples of non-function. Whereas 100 (28.4%) and 122 (34.7%) students' provided wrong examples for function and non-function respectively, while the rest students missed to provide example for function and non-function. Majority of the students who missed the definition of a function failed to give examples of function and non-function.

From the above results one can observe that all students' defined function correctly provides examples of function and non-function correctly. Whereas, most of the students who didn't defined function clearly, difficulty in providing example for function and non-function. This indicated that students have difficulty on the definition of function. For instance, most of their wrong definition of obtained from students responses are: function is like an equation which has variable x and y, function is a relation between two variables and function is an ordered pair. These definitions were incomplete definition for functions with necessary parts missing. This result is consistent with [13] findings.

Table 2: Summary of respondents to item 2 and 3 (see appendix B)

Item	Alternative												Total	
	a		b		C		d		e		f			
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
2	90	25.6	82*	23.3	30	8.5	150	42.6	0	0			352	100
3	42	11.9	24	6.8	18	5.1	96*	27.3	20	5.7	152	43.2	352	100

* indicates correct answer of the item.

The second approach employed to identify students misunderstanding in the concept of function is through item 2 and 3. For item 2, only 82 (23.3%) of the students selected the correct answer i.e., option (b) (see table 2). From this one can see that most students do not connect successfully the definition of function with their choice of the items. On item 3 of table 2, students were asked to identify which relation is/are a function on paper test, and as indicated and only 96 (27.3%) of the students chose the correct option (d); while the remaining 256 (72.7%) did not get the correct choice of this item. While, majority of them 152 (43.2%) chose option 'f'. This shows that as students' have difficulty to realize the differences regarding function and relation.

In line with this on oral question (item 1, (see appendix B)) students were asked to answer whether a relation is a function or not and then to justify their answers. The responses of students are given below:

S1: No, it depends on the value of the first element, if the first element is repeated the relation is not representing a function.

S2 and S4: Yes, since a relation there is correspondence between them hence they form a function.

S3: No, it depends on the values of the domain and range, if they are duplicated the relation is not representing a function.

S5 and S6: 'Yes, any relation can form function'.

Furthermore, as indicated by teachers on questionnaire item number 1 (see appendix C) responded as students hardly identified function and relation. Hence, from test, interview and teachers questionnaire, some examples of misconception about function was reflected as 'any relation is a function'.

4.2 Students' ability to describe functions in different representation

There are different common ways to represent functions, including verbal, graphical, tabular, mapping, and algebraic representation. Meaningful understanding the concept of function requires students to construct multiple representations as well as operations for transforming from one representation to another.

VERBAL REPRESENTATION OF FUNCTION

To explore student's conception of verbal representation of function the following three yes/no questions with justification were asked.

As shown in table 3, of students answered for item 1, 190 (54%) of students were not considered the item as a function. While 146 (41.5%) of them correctly indicated item 1 as a function but with either wrong justification or without justification. Regarding item 2, the majority of the students, 188 (53.4%), considered the item as a function. But, was only 4 (2.1%) of them provided the correct justification. On item 3, 138 (39.2%) of the students correctly answered the item where, only 8 (5.8%) of them provided correct explanation for their answer.

According to the above results, most of the respondents failed to give correct answer for the items. Even, the majority of the students who answered correctly were unable to provide supportive justification. This indicates that majority of the students had difficulty to represent functions verbally. This study is consistent with [29].

Moreover, the result obtained from students oral item number 2 (see appendix B) and teachers questionnaire question number 3 (see appendix C) indicates that the students have difficulty to understand verbal methods of describing function due to students mathematics background and language problem.

Table 3: Summary of Students' response to verbal representation of function (see appendix A)

Item	Item	Answer				N	%
4	Is there a function that maps every number different from zero to its square and maps 0 to 1	Correct	CJ	0	0%	146	41.5
			WJ	70	47.9%		
			NJ	76	52.1%		
			Total	146	100%		
		Wrong				190	54
		Missing				16	4.5
		Total				352	100
5	Are there functions all of whose values are equal to each other?	Correct	CJ	4	2.1%	188	53.4
			WJ	78	41.5%		
			NJ	106	56.4%		
			Total	188	100%		
		Wrong				150	42.6
		Missing				14	4
		Total				352	100
6	Is there a function that corresponds 1 to each positive number, -1 to each negative number, and 0 to 0?	Correct	CJ	8	5.8%	138	39.2
			WJ	36	26.1%		
			NJ	94	68.1%		
			Total	138	100%		
		Wrong				200	56.8
		Missing				14	4
		Total				352	100

Note: CJ=Correct Justification; WJ=Wrong justification and NJ=No justification

GRAPHICAL REPRESENTATION OF FUNCTION

In order to investigate students' understanding regarding the graphical representation of function, three questions

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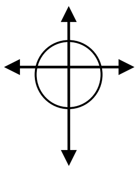
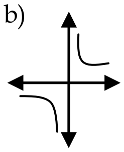
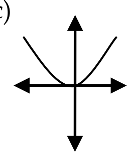
related to the graph of functions were asked (see table 4).

Table 4: Summary of Students' response to graphical representation of function (see appendix A)

Item number 7	Answer				n	%
a) 	Correct	CJ	22	16.9%	130	36.9
		WJ	32	24.6%		
		NJ	76	58.5%		
		Total	130	100%		
	Wrong				206	58.5
	Missing				16	4.6
	Total				352	100
b) 	Correct	CJ	16	11.8%	136	38.6
		WJ	34	25%		
		NJ	86	63.2%		
		Total	136	100%		
	Wrong				210	59.7
	Missing				6	1.7
	Total				352	100
c) 	Correct	CJ	24	12.7%	188	53.4
		WJ	46	24.5%		
		NJ	118	62.8%		
		Total	188	100%		
	Wrong				146	41.5
	Missing				18	5.1
	Total				352	100

Note: CJ=Correct Justification; WJ=Wrong justification and NJ=No justification

In order to investigate students' understanding regarding the graphical representation of function, three questions related to the graph of functions were asked (see table 4). In the first question, 130 (36.9%) of the students correctly indicated that the graph did not represent a function. Out of these students, only 22 (16.9%) of them provided correct justification. The second item was correctly answered by 68 (38.6%) of students, but correct reasoning was done by only 16 (11.8%) of the students. The last question was correctly answered by major students, 188 (53.4%), but only 24 (12.8%) of them provided correct justification.

The responses obtained from students' shows that most of the students missed to categorize the graphs as a function or not and they missed to justify their answers. Even those answered the question correctly have difficulties to provide reason. Moreover, those students provided correct justification for their answer used the vertical line test to determine. However, implementation of the vertical line test does not mean that the students understand it is a test for correspondence, or if they think of it as an easy classification method [13]. Most of students wrongly justified as if the graph is continues it represent a function while discontinuous graph is not a function.

In line with this, students were interviewed how they determine whether certain graphs of relations represent functions or non-functions. Accordingly, excerpts from responses to oral question 5 (see appendix C) are given as follows:

S1 and S2: We were used the vertical-line test to determine whether a graph is a function or not, if the vertical-line crosses the graph at one point then it is a function by definition.

S3 and S4: If the x-axis is crossed more than once, the graph is not a function.

S5 and S6: If the graph is continuous then it represents a function.

The interview results also indicated that majority of the students seem to have difficulties and their ideas of graphical representation of function were not coherent ideas.

In general, when we analyze the test and the interview results, students' misconceptions about the graphical representation of functions were explored. For example, several students claimed that the graph of a function must be continuous and discontinuous graph is not representing a function. Moreover, the following responses of students help the researcher to understand students' problem of understanding the graphical representation of function. Some of the incorrect responses are:

- If the graph crosses the x-axis more than once, the graph cannot be a function.
- The graph of function must cross x and y axis
- Most students did not consider a graph of a non-continuous function as a function. This study result is consistency with [23], [24] results.

TABULAR REPRESENTATION OF FUNCTION

To explore students` understanding about tabular representation of function, three questions were asked. The first question, was answered by 172 (48.9%) of the students correctly (but correct reasoning was done by only 20 (11.6%) of them). In the second question, 178 (50.6%) of the students were answered the item correctly, but correct justification was done by only 22 (12.4%) of them. The last question was responded by 130 (36.9%) of the students correctly. But out of these, correct justification was made by only 12 (9.2%) of the students (table 5).

Table 5: Summary of students' response to tabular representation of function (see appendix A)

Item number 8	Answer				n	%
a)	Cor	CJ	20	11.6%	172	48.9

X Y -2 4 -1 4 0 4 1 4 2 4	rect	WJ	40	23.3%		
		NJ	112	65.1%		
		Total	172	100%		
	Wrong				148	42
	Missing				32	9.1
	Total				352	100
b) X Y 2 -2 2 -1 2 0 2 1 2 2	Correct	CJ	22	12.4%	178	50.6
		WJ	44	24.7%		
		NJ	112	62.9%		
		Total	178	100%		
	Wrong				142	40.3
	Missing				32	9.1
	Total				352	100
c) X Y -4 1 -3 2 -4 4 -3 5 0 6	Correct	CJ	12	9.2%	130	36.9
		WJ	36	27.7%		
		NJ	82	63.1%		
		Total	130	100%		
	Wrong				182	51.7
	Missing				40	11.4
	Total				352	100

Note: CJ=Correct Justification; WJ=Wrong justification and NJ=No justification

From this one can see that, students also have difficulty on tabular representation of function which was reflected by their miss justification that "all x-values are different". They also claimed that if x-values are repeated they cannot be considered it as a function without observing the value of y. This result was consistent with [34] study result.

ALGEBRAIC REPRESENTATION OF FUNCTION

From table 6 majority of students 194 (55.1%) and 184 (52.3%) were correctly identified the algebraic expressions as a function and non-function for items a and c respectively. Whereas, 138 (39.2%) and 114 (32.4%) were correctly identified the algebraic expressions as a function and non-function for items b and d respectively. Of the students answered the question correctly only 2 (1.4%), 2 (1.4%) and 22 (12%) of them for items a, b and c respectively provided correct justification, while none of them provided reason for item d.

Table 6: Students' response to algebraic representation of function (See appendix A)

Item number 9	Answer				N	%
a) $2x+y=5$	Correct	CJ	2	1.4%	194	55.1
		WJ	46	23.6%		
		NJ	146	75%		
		Total	194	100%		
	Wrong				114	32.4
	Missing				44	12.5
	Total				352	100
b) $5x+3=0$	Correct	CJ	2	1.4%	138	39.2
		WJ	40	29%		
		NJ	96	69.6%		
		Total	138	100%		
	Wrong				176	50
	Missing				38	10.8
	Total				352	100
c) $4y+1=0$	Correct	CJ	22	12%	184	52.3
		WJ	38	20.7%		
		NJ	124	67.4%		
		Total	184	100%		
	Wrong				144	40.9
	Missing				24	6.8
	Total				352	100
d) $x^2+y^2=25$	Correct	CJ	0	0%	114	32.4
		WJ	24	21.1%		
		NJ	90	78.9%		
		Total	114	100%		
	Wrong				176	50
	Missing				38	10.8
	Total				352	100

Note: CJ=Correct Justification; WJ=Wrong justification and NJ=No justification

From the above result, one can observe that the majority of the students have also difficulty to identify algebraic form of function. For examples a majority of the students claimed that $x^2+y^2=25$ represented a function and $5x+3=0$ is not a function. Moreover, most of the teachers participated in this study (see appendix C) were pointed out that students often believe that a function must include both x and y in the expression.

From above results and students justifications one can observe that: if variables x and y included in the expression then it represents a function and if one of the variables not included in the expiration then the given equation does not represent function. This finding is consistent with study results of [23], [30].

5 CONCLUSIONS

Students possess a variety of difficulties, misconceptions and inaccurate concept images about the function concept. Students' English language skills, students' concept of function background weakness and difficulties of clear understanding of the definition of function are the major limiting reasons. Further, once a student has constructed a concept image, they no longer refer to the concept definition.

Although, the concept of function is central to understanding mathematics, yet students' understanding of functions appears either to be too narrowly focused or to include wrong assumptions. Thus, the following recommendations are made as a result of the findings in this investigation:

- The teacher should discuses deeply the definition of function, the different ways to represent, functions and the connections between the two with students' by arrange additional tutorial classes to succeed in teaching concepts of functions.
- Teacher ought to become aware of their students' understanding and possible misconceptions on the concept of function by referring to literatures in mathematics education and they would take measure to improve the problem.
- Students should give attention for the function lessons which help them for their future experience of mathematical concepts.
- The method used to analyze the data was descriptive statistics such as frequency and percentage. So it is advisable for further research to use other descriptive and inferential statistics.

After carrying out this study the researcher believe that some areas need more research like the attitude of students toward mathematics, pressure of learning subject using mother tongue up to grade eight on education in particular on mathematics and impact of teaching learning methods on students understanding of the concepts of function.

6 Appendices

APPENDIX A

Mathematics test for Grade nine Students

School Name: ________________________student's Name: ___________ Section: ______ Roll No. __________ Sex: ___________ Age: ________

Note that the general objective of grade nine mathematics courses on the topic function is students should be develop basic knowledge about function. Hence based on this the following items were designed.

Item 1: Explain the following questions.

1.1 Define the concept of function as you understand.

____

____

1.2 Give at least one example of functions.

____

____

1.3 Give at least one example of non-function (which is not a function).

____

____

Item 2: Which of the following is true?

a) Function is a correspondence between two sets that assigns to every element in the first set exactly one element in the second set.

b) There exist functions all of whose values are equal to each other.

c) There exist functions all of whose first elements are equal to each other. d) a and b e) all are correct

Item 3: Which of the following relation is/are a function?

a) {(1,3), (-2,0), (0,-2) } b) {(1,3), (-2,3), (0,-2) }
c) {(1,3), (-2,0), (-2,3) } d) a and b e) a and c f) all

Item 4: Is there a function that maps every number different from zero to its square and maps 0 to 1? a) Yes b) No

Explanation:____

Item 5: Are there exist functions all of whose values are equal to each other?

A) Yes b) No

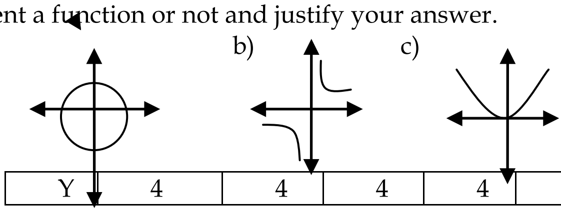
Explanation:_____________________________________

Item 6: Is there a function that corresponds 1 to each positive number, corresponds -1 to each negative number, and corresponds 0 to 0? A) Yes b) No

Explanation:____

Item 7: Examine whether each of the following graphs represent a function or not and justify your answer.

a) b) c)



____

Item 8: Do the following tables represent functions? Justify your answer in the space provided bellow.

a)

X	-2	-1	0	1	2
Y	4	4	4	4	4

b)

X	2	2	2	2	2
Y	-2	-1	0	1	2

c)

X	-4	-3	-4	-3	0
Y	1	2	4	5	6

Item 9: Determine if the following equations represent functions or not and justify your answers.

a) $2x + y = 5$ ______

b) $5x + 3 = 0$ ______

c) $4y + 1 = 0$ ______

d) $x^2 + y^2 = 25$ ______

Appendix B

Oral questions for grade nine students

Part I: The following questions were developed based to identify students' difficulties and misunderstanding on the concept of function.

1. Is a relation a function or not and justify your answer?
2. What are factors that affect you to develop the concept of function well?
3. How you determine whether the graphs relation represent a functions or non-functions?

Appendix C

Questionnaire for Mathematics teachers of Grade 9

The purpose of this questionnaire is to collect information about the current practices and challenges of grade nine students to understand the concept of function in west showa zone. The information to be obtained through this questionnaire will be used only for academic research undertaking. Thus, you are kindly requested to give your response genuinely and frankly on the basis of the questionnaire. Your cooperation is highly valuable to complete the study.

Thank you in advance for your cooperation.

General Direction

No need of writing your name on the questionnaire. For question that require your opinion or comments please write your response on the space provided.

Name of the School: ________, Teaching experience _____

Qualification ________, Sex _______, and Age _________

1. What are the difficulties of students to learn concept of function?
______
______
2. How is your student's understanding on the concept of function
______
______
3. In your opinion what are students misunderstanding of concept of function?
______
______

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